To this end, we consider the symplectic group

$$G = \mathrm{Sp}(2n) = \left\{ g = \begin{pmatrix} a & b \\ c & d \end{pmatrix},\ a,b,c,d \in \mathrm{Mat}_n(\mathbb{R}), g'Jg = J \right\}, \qquad J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}.$$

(Here g' stands for the matrix transposed to g and I is the identity matrix.) The Weil representation $T(g)$, $g \in G$, acts on the space $L_2(\mathbb{R}^n)$ by the relations [2, pp. 61–62]

$$(T(g_b)f)(x) = \exp(\pi i(bx,x)), \quad g_b = \begin{pmatrix} I & b \\ 0 & I \end{pmatrix},\ b = b', \qquad (T(J)f)(x) = \widehat{f}(x).$$

Conditions (3) are equivalent to the relations $f_2 = T(g_a)f_1$, $T(J)f_2 = \lambda T(g_b)T(J)f_1$. Hence, $T(J^{-1}g_{-b}Jg_a)f_1 = \lambda_1 f_1$, where $|\lambda_1| = 1$. We set $h_{a,b} = J^{-1}g_{-b}Jg_a$. Then we have

$$h_{a,b} = \begin{pmatrix} I & a \\ b & I + ba \end{pmatrix}, \qquad T(h_{a,b})f_1 = \lambda_1 f_1.$$

Thus, f_1 is an eigenvector of $T(h_{a,b})$. This is possible only if the matrix $h_{a,b}$ is conjugate (in the group G) to an orthogonal matrix of the form

$$\begin{pmatrix} c & d \\ -d & c \end{pmatrix}, \qquad c,d \in \mathrm{Mat}_n(\mathbb{R}),\ c + id \in U(n).$$

The complete characterization of the pairs of symmetric matrices a, b for which these properties are satisfied is unknown to the author. We can readily show that this condition holds for $a > 0$ and $-4a^{-1} < b < 0$ (the matrix inequalities are defined by means of quadratic forms). In this sufficient condition we can interchange the matrices a and b as well. Under these conditions, there exists a pair f_1, f_2 satisfying (3). For $n = 1$, this pair can be written in the form (2). We omit the explicit formulas for functions f_1, f_2 from (3). We only note that these formulas involve Hermite's polynomials and exponential functions.

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Absolute, Perfect, and Unconditional Convergence of Double Series*

M. I. Kadets

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A series $\sum x_n$ in a Banach space X is said to be *absolutely convergent* if the number series $\sum \|x_n\|$ converges, *perfectly convergent* if the series $\sum \alpha_n x_n$ converges for any coefficients $\alpha_n = \pm 1$ (in this case we write $\alpha \in D$), and *unconditionally convergent* if the series $\sum x_{\pi(n)}$ is convergent for any permutation $\pi\colon \mathbb{N} \to \mathbb{N}$ of positive integers. The notions of perfect and unconditional convergence are equivalent. Each absolutely convergent series converges perfectly; the converse is true if and only if X is finite-dimensional. In the following, we use the Banach space $\mathrm{Perf}(X)$ of perfectly convergent series in X equipped with the norm $\sup\{\|\sum \alpha_n x_n\| : \alpha \in D\}$. For these notions and facts, see [1].

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Let us consider double series. First, we give a definition of convergence for a double series; this definition is somewhat stronger than the Pringsheim convergence.

Definition 1. A series $\sum x_{ij}$ in a Banach space X converges to $s \in X$ if (a) each column series $\sum_i x_{ij}$, $j \in \mathbb{N}$, converges, (b) each row series $\sum_j x_{ij}$, $i \in \mathbb{N}$, converges, and (c) the sequence of "rectangular" partial sums converges to s:

$$s = \lim s_{mn} = \lim \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \colon \min(m,n) \to \infty \right\}.$$

The notions of absolute, perfect, and unconditional convergence for double series, which are well-adapted to the structure of these series, can be introduced as follows.

Definition 2. A series $\sum x_{ij}$ is said to be *absolutely convergent* if the number series $\sum \|x_{ij}\|$ converges.

Definition 3. A series $\sum x_{ij}$ is said to be *perfectly convergent* if for any $\alpha, \beta \in D$ the series $\sum x_{ij}\alpha_i\beta_j$ converges.

Definition 4. The series $\sum x_{ij}$ is said to be *unconditionally convergent* if for any permutations π and σ of positive integers the series $\sum x_{\pi(i),\sigma(j)}$ converges.

The following statement characterizes perfect convergence in terms of linear operators.

Theorem 1. *Consider a double series $\sum x_{ij}$ in a Banach space X. The following conditions are equivalent*: (A) *the series is perfectly convergent*; (B) *the vector-valued matrix $A = (x_{ij})$ generates a compact linear operator $A\colon c_0 \to \mathrm{Perf}(X)$ by the formula $Ae_j = \{x_{ij}\}_{i=1}^{\infty}$, $j \in \mathbb{N}$, where (e_j) is the canonical basis of the space c_0. The norm of this operator is given by*

$$M(A) = \sup \left\{ \left\| \sum x_{ij}\alpha_i\beta_j \right\| : \alpha, \beta \in D \right\}. \tag{1}$$

The following two theorems describe the relationship between perfect, absolute, and unconditional convergence for double series.

Theorem 2. *A double series is perfectly convergent if and only if it is unconditionally convergent.*

Theorem 3. *There exist double number series that converge perfectly but not absolutely.*

Sketch of proof. We construct the desired counterexample using "blocks" obtained by normalizing Walsh matrices appropriately. Recall that the Walsh matrix W_n, $n \in \mathbb{N}$, is an orthogonal $m \times m$ matrix, $m = 2^n$, all of whose entries are equal to ± 1. The sequence of Walsh matrices can be defined inductively as follows:

$$W_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad W_2 = \begin{pmatrix} W_1 & W_1 \\ W_1 & -W_1 \end{pmatrix}, \quad \text{etc.}$$

The sum of absolute values of the entries of W_n is $N(W_n) = m^2$. The functional (1) satisfies the inequality $M(W_n) \le m^{3/2}$. Let us introduce the matrices $A_n = m^{-2}W_n$. Clearly, $N(A_n) = 1$ and $M(A_n) \le 1/\sqrt{m}$. Consider the infinite block diagonal matrix $A = (a_{ij})$ with diagonal blocks A_n. The double series corresponding to this matrix converges perfectly ($M(A) \le \sqrt{2} + 1$), but not absolutely ($N(A) = \infty$).

Remark 1. Theorems 1 and 3 remain valid if we reject conditions (a) and (b) in Definition 1. However, Theorem 2 fails in this case. A counterexample is given by the double series $\sum a_{ij}$, where

$$a_{1j} = (-1)^j, \; a_{2j} = -a_{1j}, \; a_{ij} = 0 \quad \text{for } i > 2, \; j \in \mathbb{N}.$$

Remark 2. Although the notions of unconditional and absolute convergence of double series are different, the following statement is valid: if a series $\sum a_{ij}$ is unconditionally convergent, then the series $\sum a_{ij}^2$ converges.

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On the Deformation Quantization, on a Kähler Manifold, Associated with a Berezin Quantization

A. V. Karabegov

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In recent years, a series of papers [1–4] appeared in which the deformation quantization, on certain classes of Kähler manifolds, arising from a Berezin quantization [5] was studied. On an arbitrary symplectic manifold, the deformation quantization introduced in [6] can be constructed in many different ways [7–9]. In the present note, we give the construction of a deformation quantization on an arbitrary Kähler manifold by using Fedosov's approach [9] and indicate the relationship between this quantization and a Berezin one. For all cases considered in [1–4], the deformation quantization coincides with that constructed in the present paper.

1. Let us give the necessary definitions. A deformation quantization on a symplectic manifold M is the structure of an associative algebra in the space of formal power series $\mathcal{F} = C^\infty(M)[[\nu]]$ in which the multiplication $\star$ is given by a sequence of bidifferential operators $\{C_r(\cdot\,,\cdot)\}$, $r = 0, 1, \dots$; namely, for $f, g \in \mathcal{F}$ we set $f \star g = \sum_{r=0}^\infty \nu^r C_r(f, g)$, where

$$C_0(f,g) = fg, \qquad C_1(f,g) - C_1(g,f) = i\{f,g\}, \tag{1}$$

and $\{\cdot\,,\cdot\}$ is the Poisson bracket on M which is naturally extended to $\mathcal{F}$.

A (special) Berezin quantization on M is given by a family of associative algebras $\mathcal{A}_\hbar \subset C^\infty(M)$, where the parameter $\hbar$ ranges over a set E of positive reals with limit point 0. Then in the product $\prod_{\hbar\in E} \mathcal{A}_\hbar$, with component-wise product $*$, one chooses a subalgebra $\mathfrak{U}$ such that for an arbitrary element $f = f(\hbar) \in \mathfrak{U}$, where $f(\hbar) \in \mathcal{A}_\hbar$, there exists a limit $\lim_{\hbar\to 0} f(\hbar) = \varphi(f) \in C^\infty(M)$. The following correspondence principle must hold: for $f, g \in \mathfrak{U}$

$$\varphi(f * g) = \varphi(f)\varphi(g), \qquad \varphi(\hbar^{-1}(f * g - g * f)) = i\{\varphi(f), \varphi(g)\}. \tag{2}$$

We can readily extend the Berezin definition of quantization to associate with it a deformation quantization. Assume that the algebra $\mathfrak{U}$ is chosen in such a way that its elements $f = f(\hbar)$ can be expanded in asymptotic series with respect to $\hbar \to 0$, with coefficients from $C^\infty(M)$, $f \sim \sum_{r=0}^\infty \hbar^r f_r$. Define a mapping $\psi\colon \mathfrak{U} \to \mathcal{F}$ by setting $\psi(f) = \sum_{r=0}^\infty \nu^r f_r$. We say that the Berezin quantization is associated with a deformation quantization (i.e., with the structure of an algebra) on $\mathcal{F}$ if ψ is an algebra homomorphism. In this case, the correspondence principle (2) follows from (1).

2. Let us now define a deformation quantization on an arbitrary Kähler manifold. Let M be a Kähler manifold, let $\dim_{\mathbb{R}} M = 2m$, and let ω be a Kähler form on M. For any open subset $U \subset M$ we write $\mathcal{F}(U) = C^\infty(U)[[\nu]]$. On $\mathcal{F}(U)$, there is an action of the formal series of differential operators. Let U be a coordinate chart with coordinates $z = (z_1, \dots, z_m)$ on which a potential $\Phi = \Phi(z, \bar z)$ of the Kähler form

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