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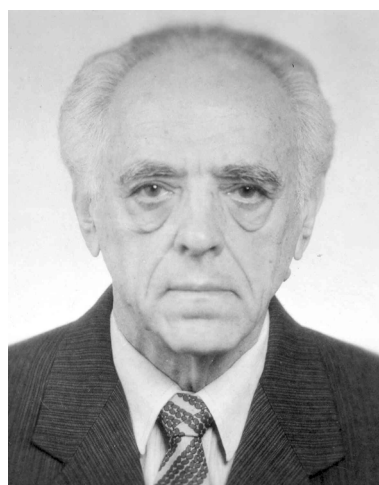
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Mikhail Iosifovich Kadets (on his 80th birthday)

Mikhail Iosifovich Kadets celebrated his 80th birthday on 30 November 2003.

Kadets was born in Kiev. He finished secondary school a few days before the beginning of the Second World War, and in 1943 he was inducted into the Soviet Army. After demobilization in 1946 he enrolled at Khar'kov University. Under the influence of his mathematical analysis teacher, V. K. Baltaga, he became interested in extending Riemann's classical theorem on conditionally convergent series to the case of finite- and infinite-dimensional normed linear spaces. An important occurrence for him was the publication in 1948 of the Ukrainian translation of Banach's book, *Théorie des opérations linéaires*. These two influences largely determined the course of Kadets' research interests.



After graduating from the university in 1950, Kadets went to work in the town of Makeevka, in Donetsk Province. Here, far from centres of mathematics, his 'research advisor' was Banach's book. Many open problems were formulated in this book.

The first problem that attracted his interest was the problem of the topological equivalence of infinite-dimensional Banach spaces, in particular, the question of whether the spaces c_0 and $C[0, 1]$ are homeomorphic, and the same question for the spaces ℓ_1 and $C[0, 1]$. He began by trying to prove that c_0 and ℓ_1 are homeomorphic. At that time the apparatus of non-linear functional analysis had not yet been sufficiently developed. The standard method, which had established that the spaces ℓ_p are homeomorphic for $1 \leq p < \infty$, did not work. To solve the problem of whether c_0 and ℓ_1 are homeomorphic, Kadets employed the apparatus of best approximations. It turned out not to be possible to use this tool directly, because the sequence of deviations of a given element in the canonical norm of c_0 was unsuitable for establishing the one-to-oneness of the maps. To overcome this obstacle, Kadets introduced in c_0 a new norm of Orlicz type and proved that c_0 and ℓ_1 are indeed homeomorphic by using the sequence of deviations of a given

element from the subspaces spanned by the first n vectors of the unit vector basis in this new norm.

In the following years Kadets established that various classes of Banach spaces are homeomorphic. Finally, in 1966 he proved that all infinite-dimensional separable Banach spaces are homeomorphic. The solution of this problem of Banach and Fréchet was undoubtedly one of the most important moments in the history of the development of the theory of Banach spaces.

The apparatus that Kadets developed in solving this problem has turned out to be very important. He originated two methods, called the method of coordinates and the method of equivalent norms. Roughly speaking, the method of coordinates consists in the fact that if X and Y are two Banach spaces with bases $\{x_n\}$ and $\{y_n\}$, then a homeomorphism can be established in such a way that for any subset M of the natural numbers the subspace spanned by $\{x_n : n \in M\}$ is carried onto the subspace spanned by $\{y_n : n \in M\}$.

The second method he developed, the method of equivalent norms, consists in the following. In a given Banach space an equivalent norm is introduced that has properties close to those of a Hilbert space norm. For example, to construct a homeomorphism he introduced an equivalent locally uniformly convex norm. This norm has two important properties of a Hilbert sphere: first, the sphere does not contain segments, and second, the weak topology and norm topology coincide on the sphere. The second property of the sphere later came to be called the Kadets property.

At present the theory of renormings and its applications has turned into a separate branch of the theory of Banach spaces. Significant parts of some monographs are devoted to this theory: M. M. Day, *Normed linear spaces*, Springer-Verlag, 1973; J. Diestel, *Geometry of Banach spaces*, Springer-Verlag, 1975. The monograph *Smoothness and renormings in Banach spaces* by R. Deville, G. Godefroy, and V. Zizler (Wiley, 1993) is devoted entirely to the theory of renormings. We remark also that a class of non-linear maps called ‘sigma-cutoff continuous’ maps was recently introduced. In essence, this class goes back to the homeomorphism constructed by Kadets. Kadets himself used the method of equivalent norms to prove that each separable space has a non-linear operator basis.

While solving the homeomorphism problem, Kadets also worked on other important questions in the geometry of Banach spaces. He found the first application of the Bishop–Phelps theorem on linear functionals that attain their norms: namely, he showed that a separable Banach space admits a Fréchet-differentiable norm if and only if its dual space is separable. We remark that this characterization played an important role in Asplund’s study of Fréchet-differentiability of continuous convex functions on a Banach space. The question of characterizing Banach spaces admitting a Fréchet-differentiable norm in the general (non-separable) case remains open.

Kadets found an elegant necessary condition for the unconditional convergence of series in terms of the modulus of convexity of a Banach space. This condition is a predecessor of the condition of non-triviality of the cotype of a Banach space. Kadets’ characterization of Schauder bases and Cesàro bases in terms of their multipliers is well known.

Kadets studied questions involving subspaces of L_p , and in the process filled some gaps in the table on linear dimensions in Banach's book. He found asymptotically sharp estimates for the modulus of convexity in the spaces L_p , $1 < p \leq 2$.

These results put Kadets among the founders of the modern theory of Banach spaces. In the 1960s he began to create a school of the theory of Banach spaces in Khar'kov. Results coming from this school have become widely known. In particular, Day's book lists the Khar'kov journal *Teoriya Funktsii, Funktsional'nyi Analiz i ikh Prilozheniya* (*Theory of Functions, Functional Analysis and their Applications*) among the five basic journals publishing papers on the theory of Banach spaces.

Kadets later broadened the circle of his interests. Investigating almost periodic functions with values in Banach spaces, he showed that the class of spaces not containing subspaces isomorphic to c_0 is the maximal class of Banach spaces in which the Bohl–Bohr theorem on indefinite integrals of almost periodic functions with values in a Banach space is valid.

Working with his student M. G. Snobar, Kadets found an asymptotically sharp estimate for the projection constants of finite-dimensional normed spaces. This estimate and the estimates of V. I. Gurarii, M. I. Kadets, and V. I. Matsaev for the Banach–Mazur distances between certain classical finite-dimensional spaces constituted some of the pioneering results in the local theory of Banach spaces.

Of later results we mention the theorem that a strictly convex Banach space with the H -property that does not contain ℓ_1 is a symmetrically locally uniformly convex space. This elegant result is a theorem of a new type, connecting geometric properties with topological properties. A generalization of this theorem lies at the basis of the characterization of Banach spaces admitting a locally uniformly convex renorming.

As mentioned above, Kadets studied the question of extending Riemann's theorem on conditionally convergent series to the case of Banach spaces already in his student years. In 1989 he and K. Wozniakowski proved that in each infinite-dimensional Banach space there is a series whose domain of sums consists of two points.

Kadets also obtained important results lying outside the scope of the theory of Banach spaces: namely, in approximation theory and harmonic analysis. He found a refinement of Chebyshev's alternation theorem for the polynomial of best approximation. His ' $\frac{1}{4}$ Theorem' is well known: the proof that $\frac{1}{4}$ is the best constant in the Paley–Wiener theorem on non-harmonic series.

His paper with A. Pełczyński, "Bases, lacunary sequences and complemented subspaces in the spaces L_p ", is one of the best known and most often cited papers in the theory of Banach spaces. In it the authors used the sets M_ε^p introduced by Kadets in one of his early papers to develop a technique enabling them to obtain many fundamentally new results on the isomorphic structure of subspaces of L_p . Some of these results have numerous applications in approximation theory.

Kadets has devoted much time and effort to his pedagogical work. As already mentioned, he created a school on the theory of Banach spaces in Khar'kov. The centre of this school is a continually active seminar at the Khar'kov State Academy of the Municipal Economy, where Kadets has worked from 1965 to the present time.

For many years he not only taught a functional analysis course at Khar'kov University, but also gave a number of special courses in the theory of Banach spaces: "Series in Banach spaces", "Biorthogonal systems and bases", "Theory of renormings", and others. His style of lecturing is a disciplined, keen, and concise language on a background of meticulous techniques of analysis and geometry. Together with V. M. Kadets he wrote the monograph *Series in Banach spaces: conditional and unconditional convergence* (Birkhäuser, Basel, 1997), which is the most authoritative source of information about the theory of series in Banach spaces. He has had many students: nineteen of them have defended PhD dissertations, and seven have become doctors of the sciences. At present his students are working in many countries of the world: Bulgaria, Germany, Israel, Spain, Russia, Syria, the USA, and Ukraine. Mikhail Iosifovich Kadets is an Honoured Scientist of Ukraine.

We wish Mikhail Iosifovich good health and many years of creative activity.

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